Journal of Aircraft
Vol. 47, No. 5, September–October 2010

Design Forum

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Improved Fuel Capacity Estimation Method

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DOI: 10.2514/1.C000270

In this paper a new semi-analytical fuel tank capacity estimation method is developed for use in the conceptual design optimization of transport aircraft. It is based on a simplified spanwise thickness-to-chord ratio distribution model. Three variables are required to define this model, but reasonable simplifications can be made to use only one. The new method accounts for different geometries of the inboard and outboard integral tanks as well as the center wing tank. Rigorous statistical tests demonstrate the new method's superiority in both relative accuracy and consistency to all the other methods tested. The 95% confidence interval of mean prediction error of the new method is −5.96 to +2.45% for conventional short- to medium-haul transport aircraft, which compares favorably with −13.37 to +6% of a control method representing the current practice. Prediction error bandwidth observed in the sample is −6.7 to +5%. This improvement reduces the uncertainty about maximum fuel capacity and enables payload-range performance constraints to be more clearly defined.

Nomenclature

A = wing panel aspect ratio
b = wing panel span, m
b_f = fuselage maximum width, m
C = chord length, m
I = fuel tank volume index, m^3
k = regression coefficients
S = wing panel area, m^2
t = airfoil maximum thickness, m
t/c = thickness-to-chord ratio
$t/c|_{\mathrm{avg}}$ = wing panel average thickness-to-chord ratio
W_{fuel} = maximum fuel load, kg
Γ = dihedral angle, deg
η = spanwise location
λ = wing panel taper ratio
σ = standard deviation
τ = $(t/c|_t)/(t/c|_r)$, ratio of thickness-to-chord ratios at the tip and root of a wing panel

Subscripts

ctr = center (wing tank)
ibd = inboard (wing panel)
obd = outboard (wing panel)
r = root (of a wing panel)
t = tip (of a wing panel)
trap = trapezoidal main wing

Received 23 January 2010; revision received 29 June 2010; accepted for publication 29 June 2010.

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Introduction

IN RESEARCH works concerning the conceptual design optimization of transport aircraft, design specifications are usually given in the simplest format: one design mission at a fixed cruise Mach number carrying a typical complement of passengers and cargo [1,2]. Such a practice is in fact misleading, in that it ignores the wide range of commercial operations that realistic designs must cope with in order to be profitable. The determination of design mission(s) and the range within which the aircraft can operate with different combinations of payload and fuel is not a trivial matter. Obert [3] cited Dassault Mercure as an example of market failures due to inappropriate design mission specification. The amount of effort that goes into the payload-range capability of real-world transport aircraft is evident in any design case study [4,5]. In the mean time, the need to cover a wider market segment also requires any realistic new design to have adequate growth potential, so that stretched versions can be developed with minimal developmental risk. Therefore, to improve realism in conceptual design optimization, it is important to consider the issue of operational flexibility and growth potential for the baseline model of any new transport aircraft.

In Fig. 1, the payload-range curves‡ of 737-600 and 737-900 are superimposed onto that of the baseline 737-700 aircraft. The design mission of the baseline aircraft is also marked. Using this figure, the factors that determine payload-range capability of both the baseline design and the derivatives can be identified.

For designers, the part of the curve beyond point 2 is of limited interest, because at point 2 and beyond, revenue-making payload needs to be traded for modest gains in range [6]. While the range with maximum payload (point 1) is set by maximum zero-fuel weight, the maximum useful range (point 2) is limited by maximum fuel

‡Payload-range data taken from Boeing's airport planning brochure.

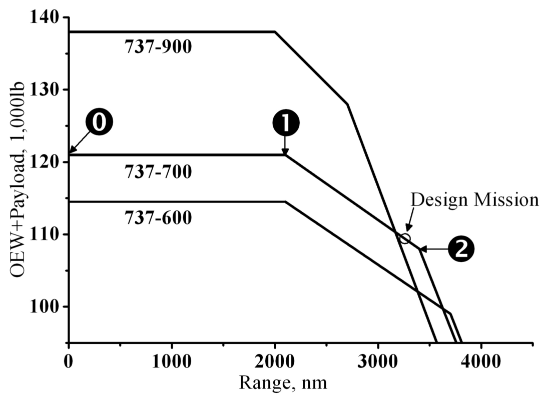

Fig. 1 Payload-range capability of 737-600, 737-700, and 737-900 (OEW denotes operating empty weight).

capacity. Maximum zero-fuel weights of the derivatives have been modified so that all versions have comparable maximum payload ranges. Sharing a common wing structure, all three versions have the same maximum fuel capacity limit [7], but the 737-900 model (with the heaviest, largest airframe, as well as payload) has the lowest average specific range by comparison. This has led the stretched version to have significantly less operational flexibility and a shorter maximum useful range than the others. It follows that fuel tank capacity is constrained by the payload-range performance of not only the baseline but also the stretched version. This is based on the assumption that a common wing design is retained for all versions, which is usually the case for development risk concerns.

In academia most treatments of payload-range performance have been simplistic so far. In most research works, constraint is only set to ensure that fuel capacity exceeds that required to fly a typical mission ([8], part II). Examples of research papers include [1,2], among others. Because optimizers can exploit underconstrained problems (see [9]), optimized designs often have the smallest fuel volume possible for the design mission. This is always a critical issue for long-range transports ([6], page 225). Perez and Behdinan [10] attempted to address this problem by specifying two different design mission requirements. In [11,12] researchers of a strut-braced wing transport also considered the need to perform an additional economic mission in the design problem. However, in none of these studies was growth potential taken into consideration. Wilson [13] studied the impact of derivatives on the baseline version. Although his conclusions about compromises that needed to be made in the baseline model are correct, Wilson assumed that any demand for additional fuel capacity was automatically met with freight hold and tailplane fuel tanks. We argue that the potential loss of revenue due to lost cargo space should be evaluated quantitatively; otherwise, the economic viability of the final design may still be dubious.

The effectiveness of any solution to the payload-range constraint problem depends first on the analysis program's ability to estimate the fuel capacity of a given external geometry. If prediction errors are such that there is considerable uncertainty about fuel capacity, any attempt to optimize with realistic payload-range and growth potential constraints would be futile. Currently, both simple formulas [6,14] and simplified fuel tank geometry models [15–17] have been used for this purpose. While the former are easy to apply but usually of only first-guess accuracy, the latter can be fine-tuned to a particular aircraft but lacks predictive ability for designs other than the one investigated. A typical formula, the Torenbeek method ([6], page 449), was said to have a prediction error in the vicinity of $\pm 10\%$ ([8], part II, page 154). However, when we tested this formula with a sample of nine transport aircraft (as shown later), its mean error was 20.99%, with a standard deviation of 11.50%. The Howe method ([14], page 132), which only uses $t/c|_{\text{avg}}$, shows a mean error of -9.19% and a standard deviation of 7.99%. Both comfortably exceed the suggested $\pm 10\%$ error bandwidth. Although a fudge factor can always be applied to force a safer, more conservative estimate to account for the uncertainty in fuel capacity, such a workaround leads to an oversized wing that requires laborious revisions later in the design process, thus negating the purpose of optimization.

To improve fuel capacity estimation, more information on the fuel tank geometry should be obtained. This can be achieved by introducing spanwise t/c distribution. Previously, a simplified t/c distribution model was proposed in [18], but no attempt was made to discuss the implementation of this model. In a brief introduction to an industry-strength aircraft synthesis program used by BAE Systems [19], it was mentioned that a t/c distribution model using tip, kink, and root t/c had been used to define wing geometry, although no additional detail about its relationship to fuel capacity estimation was given. Other attempts to account for the variation of t/c along the span were usually the by-products of using complex geometric models in conceptual design optimization. Typical examples are studies of strut-braced wings [11,12,16], in which the wing is broken down into eight strips, and both the average inboard and outboard wing panels' t/c were chosen as design variables. This increase in geometry information enabled these authors to estimate the fuel capacity of 777 with good accuracy, but their assumptions were specific to this type of aircraft, which makes its use on a wider basis questionable.

The analysis above indicates that the traditional practice of using $t/c|_{\text{avg}}$ to represent the thickness-to-chord ratio distribution of transport aircraft does not provide enough information on the external geometry to enable an accurate estimate of fuel tank capacity. We will now introduce a simplified t/c distribution model that is suitable for conceptual design optimization and then derive an improved fuel tank capacity estimation method based on this model.

Simplified Spanwise Thickness-to-Chord Ratio Distribution Model

The objective of this t/c distribution model is to introduce more information about the wing external geometry to enable a better estimate of fuel capacity. The variation of t/c along the span is represented by three characteristic thickness-to-chord ratios: $t/c|_r$,$t/c|_t$, and $t/c|_{\text{avg}}$. The latter is also used as the kink t/c. This stemmed from our observation that for all the transport aircraft for which t/c data is available, t/c at the kink is always equal to $t/c|_{\text{avg}}$ of the entire wing ([20], page 119). In actual designs the variation of t/c in intermediate airfoils is seldom strictly linear (see [3], part 4), but for our purposes we only have to assume that wing panel thickness variation is linear, which is not unreasonable considering the typical distribution illustrated in Fig. 2.

The simplified spanwise thickness-to-chord ratio distribution model is illustrated in Fig. 3. In the lower part of Fig. 3, the differences between the actual distribution (solid line), the simplified distribution model (dashed line), and the traditional approach using $t/c|_{\text{avg}}$ (dotted-dashed line) are illustrated. Note that the straight dashed lines connecting the tip, kink, and root t/c are meant to

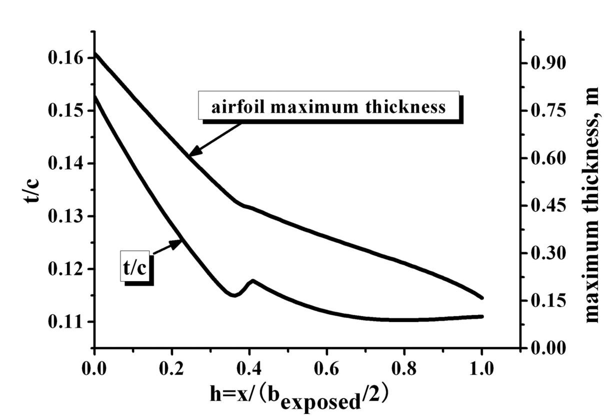

Fig. 2 Spanwise t/c and maximin thickness distribution of A320 (t/c distribution data of A320 taken from [3]).

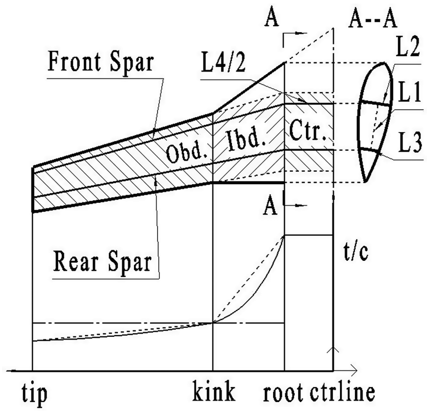

Fig. 3 Simplified t/c spanwise distribution model.

illustrate the different levels of approximation rather than a linear t/c distribution. When fuel tank volume needs to be estimated, the trapezoidal main wing planform, represented in Fig. 3 by the shaded region, is used in lieu of the actual planform. Trapezium $t/c|_{r_{\mathrm{trap}}}$ can be calculated by assuming that the trapezoidal wing has the same maximum thickness as the actual wing. This can be corroborated by checking the trapezium and actual $t/c|_r$ of A300 in [6] (page 220) and in [3] (page 258). Kink location is required to divide the wing into outboard and inboard sections. If this is not a design variable, it is acceptable to use a typical value of around 32 to 39% semispan.

All three thickness-to-chord ratios can be set as design variables in an optimization study, but if accuracy needs to be traded for efficiency, various further simplifications can be made to use one variable to define this model. Because of its strong impact on wing structural weight and fuel capacity (see [6], pages 235 and 281, [21], pages 7–15, and [3], page 574), $t/c|_r$ is recommended as a design variable. The drag divergent Mach number of the aircraft can be used to derive $t/c|_t$ via the Korn equation ([6], page 248, and [18], page 115). Finally, $t/c|_{\mathrm{avg}}$ can either be calculated from the other two thickness-to-chord ratios (assuming a linear-lofted geometry) or with an empirical relationship based on t/c distribution data of real-world aircraft (a small amount of such data is available from [6], page 220, and [20], page 119).

Improved Fuel Tank Volume Estimation Method

We now present a semi-analytical fuel tank volume estimation method based on the Torenbeek formula that makes full use of the richer information made available by the introduction of t/c distribution to maximize its predictive ability and robustness.

Derivation of Fuel Tank Capacity Formula

Observing the thickness distribution in Fig. 2, it appears that if the exposed wing planform is divided into inboard and outboard wing panels, these panels can be assumed to be linear-lofted for fuel tank estimation purposes. The Torenbeek formula can then be used to calculate the available fuel volume for each linear-lofted wing section. The center wing tank can be modeled by an extension of the inboard integral tank with no sweepback. After studying typical wing structural arrangements, we concluded that the part of the wing planform that covers the integral tanks is usually independent of the leading- and trailing-edge extensions (i.e., wing spars run straight from the wing tip to wing root) and decided to use parameters of the trapezoidal main wing planform to calculate fuel capacity. The upper part of Fig. 3 illustrates the geometry of this fuel tank model. Note that fuel tanks are confined chordwise within the space between the front and rear spars. The shaded area only represents the trapezoidal main wing planform and its extension in the wing carry-through structure, not the fuel tanks.

The resultant fuel tank model has the following form:

$$W_{\mathrm{fuel}} = W_{\mathrm{obd}} + W_{\mathrm{ibd}} + W_{\mathrm{ctr}} + W_{\mathrm{res}} \tag{1}$$

All the terms in Eq. (1) have units of kilograms instead of cubic meters to facilitate easy implementation in weight calculation. Fuel density is set to 0.803 kg/liter, consistent with that of [7]. The last term (W_{res}) does not necessarily have any physical meaning, but due to different spar locations, detailed internal structural arrangements, and so forth, we expect the formula to contain such a residual term.

It is assumed that W_{obd} and W_{ibd} both have a linear relationship with the respective wing section's fuel capacity index, defined in its general form in Eq. (2). This is identical to the original Torenbeek formula ([6], page 449), except that the original coefficient is removed:

$$I = \frac{S^2}{b} t/c|_r \frac{1 + \lambda\sqrt{\tau} + \lambda^2\tau}{(1+\lambda)^2} \tag{2}$$

In Eq. (2) geometry parameters refer to the inboard or outboard sections of the exposed trapezoidal main wing.

We also find that dihedral angle Γ varies somewhat among the aircraft included in our sample. Its effect is accounted for by dividing the measured wing panel area by $\cos\Gamma$ to find the actual area, and the same is done for wing spans. If dihedral is not a design variable, a typical value can be assumed or simply set to zero for low-wing designs.

As for the center wing tank, it is necessary to make the assumption that the wing root profiles have curves that are relatively flat between the front and rear spars. Hence, the shape of the center tank can be approximated by a uniform prism with parallelogram cross sections (see cross section A–A illustrated in Fig. 3). The width of this prism, L4, is assumed to be linearly related to fuselage width. Other linear dimensions that define the volume of this prism are L1 (distance between the spars), L2 (front spar depth), and L3 (rear spar depth). These assumptions are only valid for a group of aircraft with similar fuselage and wing designs.

According to these assumptions, L1, L2, L3, and L4 all have linear relationships with $C_{r_{\mathrm{trap}}}$, t_r, $t/c|_{r_{\mathrm{trap}}}$, and b_f. Hence, a center tank volume index assumed to have a linear relationship with the actual center tank capacity can be defined in Eq. (3):

$$I_{\mathrm{ctr}} = b_f C^2_{r_{\mathrm{trap}}} t/c|_{r_{\mathrm{trap}}} \tag{3}$$

Combining Eqs. (1–3), the improved Torenbeek method can be expressed as follows:

$$W_{\mathrm{fuel}} = k_1 I_{\mathrm{ibd}} + k_2 I_{\mathrm{obd}} + k_{\mathrm{ctr}} I_{\mathrm{ctr}} + W_{\mathrm{res}} \tag{4}$$

where k_1, k_2, and k_{ctr} are regression coefficients with dimensions of density.

With the basic formula developed, regression techniques can then be applied to find the coefficients.

Controls and Alternative Methods

Two control methods serve as bases of comparison: a refitted Howe [14] and an improved Torenbeek method without the center tank index I_{ctr} (abbreviated as w/o I_{ctr} hereafter). They are presented in Eqs. (5) and (6). The latter is otherwise identical to the improved Torenbeek method [Eq. (4)], except that it has I_{ctr} removed. They both have their original coefficients replaced with new ones regressed using our sample to avoid unfair comparisons, because the original samples used to find those coefficients were certainly different from the one used here:

$$W_{\mathrm{fuel}} = k_1 b S (t/c|_{\mathrm{avg}})(1 - 0.89\lambda + 0.49\lambda^2)/A \tag{5}$$

$$W_{\mathrm{fuel}} = k_1 I_{\mathrm{ibd}} + k_2 I_{\mathrm{obd}} + W_{\mathrm{res}} \tag{6}$$

Parameters of the reference trapezoidal main wing (in which the leading- and trailing-edge lines extend to the centerline) are in use for Eq. (5), while those of the exposed trapezoidal main wing are in use for Eq. (6).

The refitted Howe method only uses $t/c|_{\mathrm{avg}}$ and is thus completely independent of t/c distribution. The w/o I_{ctr} method makes use of

t/c distribution for the exposed trapezoidal main wing only and is meant to illustrate the significance of I_{ctr}. We expect the new method to demonstrate significant improvement in predictive ability over the controls.

Additionally, a modified Howe formula [Eq. (7)] and modified Torenbeek formula [Eq. (8)] serve as two parsimonious alternatives to the new method. They are otherwise identical to the improved Torenbeek method, except that the two wing tank indexes in Eq. (4) are replaced with the original Howe or the Torenbeek formula:

$$W_{\mathrm{fuel}} = k_1 b S (t/c|_{\mathrm{avg}})(1 - 0.89\lambda + 0.49\lambda^2)/A + k_{\mathrm{ctr}} I_{\mathrm{ctr}} + W_{\mathrm{res}} \tag{7}$$

$$W_{\mathrm{fuel}} = k_1 \frac{S^2}{b} t/c|_{r_{\mathrm{trap}}} \frac{1 + \lambda\sqrt{\tau} + \lambda^2\tau}{(1+\lambda)^2} + k_{\mathrm{ctr}} I_{\mathrm{ctr}} + W_{\mathrm{res}} \tag{8}$$

In these two cases, geometry parameters of the exposed trapezoidal main wing are used.

Regression and Analysis of Results

A sample of nine transport aircraft was chosen to find the regression coefficients and the residual terms in Eqs. (4–8). The distribution of their maximum fuel capacity versus wing area is plotted in Fig. 4. Geometric and maximum fuel capacity data were collected or measured from [7,22] and manufacturers' airport planning brochures. The size of this sample is limited by several factors. Equation (2) reveals that fuel tank capacity is very sensitive to wing area and t/c; therefore, only those with fairly accurate manufacturer's scaled drawings and authentic t/c data can be chosen. Furthermore, the sample should not contain outliers that exert unduly large influence on the regression results. This excludes those with very large wing areas such as the 707 and DC-8 from our sample. With these constraints in mind, we settled on a sample that covers a gross wing area range from 86.77 m^2 (DC-9-10) to 219 m^2 (A310), and we consider it representative of mainstream short- to medium-haul airliners.

Sample mean W_{fuel} is 21,447.956 kg, and sample standard deviation is 11,151.093 kg. It is evident that the large variation in W_{fuel} within the sample and its small size are two problems that need to be addressed before regression can be attempted.

First, because of the significant variations in W_{fuel}, ordinary linear regression (OLR) technique with which only the sum of squared residuals (an indicator of absolute accuracy) is minimized can cause those with smaller W_{fuel} to suffer unduly in terms of relative accuracy. To ensure a comparable relative accuracy for all the aircraft in this sample, a weighted linear regression (WLR) technique was used in lieu of OLR. Through trial and error, the inverse of W_{fuel} squared is found to be a satisfactory choice of weights. Average prediction error obtained using WLR technique is 2.55%, which is slightly higher than 2.3% obtained using OLR, but standard deviation of errors decreased from OLR's 3.19 to 2.95%, indicating an improvement in consistency.

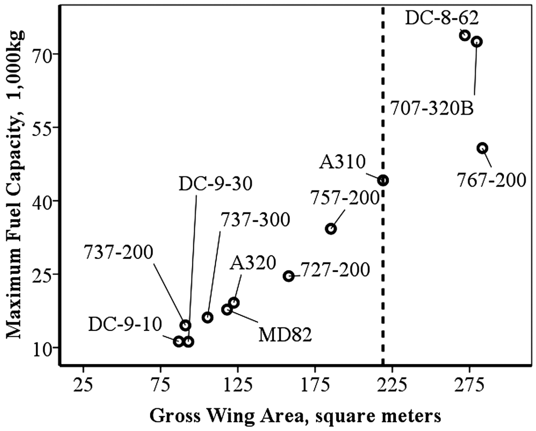

Fig. 4 Maximum fuel capacity and wing area distribution of the sample.

Second, the size of this sample is very small in a statistical context. To obtain a better measurement of the models' external validity, the jackknife method is employed to calculate the prediction errors and standard deviation of errors for all the methods. This method produces statistics that are resistant to overfitting, and for small samples it is considered more suitable than conventional linear regression techniques [23]. Comparisons between these results with those calculated normally can detect signs of overfitting.

An additional problem is that conventional linear regression techniques build on the assumption that no error exists in independent variables. However, in our case, the calculated indexes are contaminated by inaccurate manufacturers' scaled drawings, further geometric distortion due to printing the drawings, and random measurement errors, while the dependent variable W_{fuel} is affected by measurement errors on the manufacturer's side. Ideally, contributions of these errors should somehow be estimated and a more accurate regression model should be built with structural equation modeling techniques (refer to [23]). But it was found that such estimates are difficult to make, and the end results are often worse than those of WLR. Therefore, structural equation modeling was not used, and measurement errors are assumed to be absent in this paper. Although this may have resulted in slightly biased estimates of coefficients, we also argue that no empirical formula is completely free from noisy raw data. For example, in their wing weight formula study, the authors of [24] reported cases in which λ is not equal to C_t/C_r. Meticulous effort has been invested in the preparation of the data, and we believe the accuracy achieved is proof enough for the superiority of the model proposed in this paper.

In Table 1 a summary of the regression results is presented. The regression coefficients are found in Table 2. Column headings Min and Max denote prediction error lower and upper bounds. Average absolute prediction error and standard deviation of prediction errors[§] are used in conjunction with maximum prediction errors observed in the sample. These statistics indicate the overall magnitude, consistency, and bandwidth of prediction errors. More common statistics such as adjusted R^2 are not included in the final results. It is important to emphasize that because multiple correlation coefficients (adjusted R^2 and its raw version R^2) only measure the absolute amount of variations explained by a regression model, they no longer reliably capture a model's quality in terms of relative accuracy. In any case, adjusted R^2 and R^2 are higher than 0.9 for all the methods tested.

Some interesting conclusions about our fuel tank model can be drawn from the regression coefficients in Table 2. Since fuel density has been assumed to be 803 $\mathrm{kg/m^3}$, a coefficient smaller than this value would indicate that the volume represented by the corresponding individual volume index in Eq. (4) is greater than the actual volume. It can then be seen that for the improved Torenbeek method all three indexes exhibit overestimation. The magnitude of this difference is partly related to the amount of departure of actual fuel tank geometry from the assumed linear-lofting external geometry. As expected, the regression coefficients indicate that the inboard wing panels feature irregular external geometries that depart from simplified linear-lofting assumption more than the other panels. Additionally, some aircraft (737-200, for example) appear to feature kinks not only in external planform but also in front and rear spars, which will also contribute to these differences. Because the theoretical model has overestimated the tank volumes, one would expect the residual term to be negative, but it must be noted that the W_{fuel} data we collected were not accurate and can potentially alter the sign of W_{res}. For example, DC-9-30 has a larger wing than the DC-9-10 version, but the manufacturer's brochure shows that the former actually has less fuel capacity by comparison. More often than not, one version of an aircraft has multiple fuel capacity options available,

[§]Denoted by avg. |err|, average absolute prediction errors are obtained by averaging the absolute values of relative errors and are meant to represent the magnitude of relative errors. Standard deviations of prediction errors are calculated using actual signed relative errors.

Table 1 Comparison between the improved Torenbeek methods and controls

Regression method	Avg |err|	σ	Min	Max
		Refitted Howe		
N	5.42%	6.57%	−9.72%	10.43%
J	8.34%	10.47%	−16.28%	13.8%
		w/o I_{ctr}		
N	6.15%	7.60%	−8.21%	13.08%
J	7.31%	9.32%	−13.19%	10.87%
		Modified Howe		
N	3.97%	5.31%	−11.55%	6.12%
J	4.67%	5.31%	−8.66%	5.46%
		Modified Torenbeek		
N	3.81%	5.42%	−10.45%	7.91%
J	4.49%	5.63%	−9.31%	3.93%
		Improved Torenbeek		
N	2.55%	2.95%	−4.86%	4.14%
J	3.70%	4.55%	−6.68%	4.99%

Table 2 Regression coefficients

k_1, kg/m^3	k_2, kg/m^3	k_{ctr}, kg/m^3	W_{res}, kg
	Refitted Howe		
447.274	——	——	——
	w/o I_{ctr}		
375.016	614.316	——	4421.785
	Modified Howe		
247.683	——	418.030	1432.511
	Modified Torenbeek		
137.519	——	659.503	1260.096
	Improved Torenbeek		
141.799	300.753	616.412	1985.948

and such differences are not accounted for by our model. These factors have collectively led W_{res} to be positive.

Next, hypothesis testing with regards to modeling and coefficients is carried out for the regression models. All models are significant at the 0.01 level in the F-test, and all the coefficients except the residual term W_{res} passed the t-test at a 0.05 significance level. W_{res}, being a residual term to account for miscellaneous factors that the present model does not cater for, is not likely to have statistical meaning in any case. Finally, collinearity analysis calculated a condition index of 16.444, less than the threshold value of 30 (see [23], page 57).

Because we have prior knowledge of the strong physical relationship between the independent and dependent variables, tests from a statistical perspective do not provide enough information upon which to draw conclusions about the qualities of the models. We are more interested in investigating the predictive capability of the methods for aircraft within the same population other than those included in the small sample and how the introduction of t/c distribution and I_{ctr} has contributed to this capability. A comparison between statistics calculated normally (N) and those with jackknifing (J) in Table 1 indicates that all methods exhibit varying degrees of overfitting, although the improved Torenbeek method and its two alternatives did not show deterioration as dramatic as the controls. The predictive ability of these methods can be approximated by the 95% confidence interval of the respective means of prediction errors. Because the sample size is small, a Student's distribution should be used to correct for uncertainty about population mean and standard deviation [25]. Distribution of prediction errors should be checked for skewness and outliers to ensure that the conditions required for using the t confidence interval are met. For the new method, skewness of the prediction errors is 0.165, much less than two times its standard error of 0.794. The same can be said of all the other control methods, although in these cases a skewness of −0.575, close in magnitude to its standard error of 0.794, has been observed. A rule of thumb is that if skewness is found to be less than twice its standard error, then a symmetrical distribution can be assumed. We then infer that for the new method, mean prediction errors for approximately 95% of all aircraft within the population represented by the sample will be within the range of −5.96 to 2.45%. Mean prediction error confidence band is calculated for all methods in this manner and plotted in Fig. 5. The square, circle, and triangle symbols located between the T-bars indicate locations of sample mean prediction errors. In Fig. 5, Mod, Imp, and Rftd denote modified, improved, and refitted, respectively.

Three conclusions can be drawn from Fig. 5. First, the much narrower confidence interval of the new method in contrast to that of the refitted Howe method demonstrates that the introduction of t/c distribution as a whole has contributed to considerable improvement in prediction accuracy. The locations of the mean prediction errors also suggest that all but the w/o I_{ctr} method exhibit a tendency to underestimate W_{fuel}. Second, the improvement in accuracy is primarily due to the introduction of center wing tank index. This can be seen by comparing the new method with the w/o I_{ctr} method, in which the contribution of center wing tank is deliberately removed. Third, the more parsimonious alternatives exhibit levels of accuracy close to that of the improved Torenbeek method. However, this should not be interpreted as evidence that the t/c distribution model has done little to improve estimation accuracy for the wing integral tanks. It must be noted that the new method [Eq. (4)] has nearly twice as many potentially noisy parameters (three indexes and 13 independent parameters) as the modified Howe method [two indexes and eight independent parameters, see Eq. (7)], which means that the new method would suffer to a greater extent from measurement errors. The reduction in degrees of freedom, which has a more pronounced impact because of the small sample size, also suggests that greater improvement can be expected if the sample size is larger.

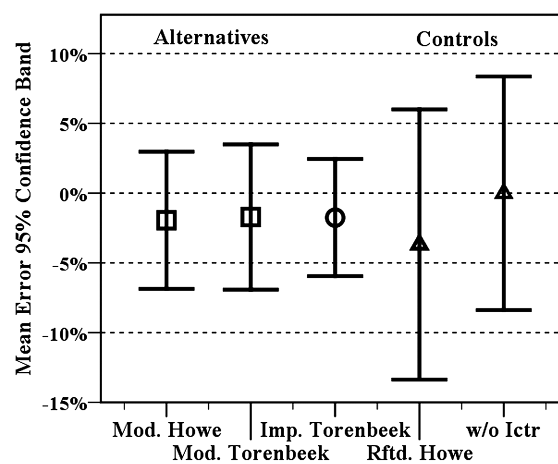

Fig. 5 Comparison of mean prediction error 95% confidence band.

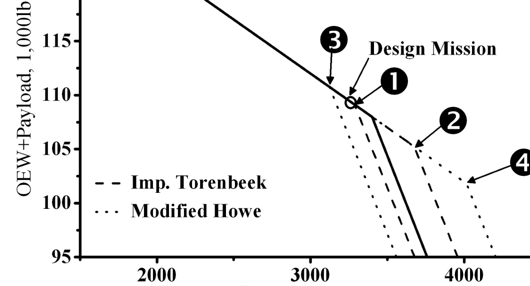

Fig. 6 Effects of fuel capacity mean prediction errors on payload-range performance.

Effects of Fuel Capacity Estimation Errors

On the payload-range diagram, changes in fuel capacity shift the maximum useful range point up or down along the curve. Overestimation of fuel capacity results in a reduction of actual maximum useful range. Uncertainty in fuel capacity due to prediction errors can then be represented by the range of the probable locations of the maximum useful range point. It follows that an inaccurate estimate leads to a greater part of the payload-range curve being uncertain. In Fig. 6 a comparison is made between the improved Torenbeek method and the modified Howe method to illustrate this effect.

Figure 6, which is resized from Fig. 1 to include only the payload-range curve of 737-700, indicates the uncertainty range due to variations in mean prediction error. Note that the uncertainty range (from point 1 to point 2) of the new method is beyond the design mission point of this aircraft, which indicates that the designer can be confident that average prediction error is small enough to ensure adequate fuel capacity. In contrast, using the modified Howe method, the uncertainty (represented by the part of the curve from point 3 to point 4 in Fig. 6) is such that there is a 25.8% probability that mean predicted fuel capacity will fall below the amount required for the design mission. This comparison clearly demonstrates that the improvement in fuel tank capacity estimation can help to establish greater confidence in optimized designs possessing required payload-range performance. Consequently, the need for time-consuming manual fine-tuning of optimized designs is minimized.

Conclusions

A competitive commercial transport aircraft needs to have adequate operational flexibility in the form of a wide variety of possible payload and range combinations. The growth potential of a baseline model is also directly dependent on the payload-range capability of the baseline design. Hence, in order to improve realism in conceptual design optimization, payload-range capability constraint must be taken into consideration.

Solutions to this problem depend on a reasonably accurate estimate of the maximum fuel capacity. It was found that this can be realized by the introduction of a simplified spanwise thickness-to-chord ratio distribution model that provides richer information about the aircraft geometry. With a few reasonable simplifications, the amount of variables needed to define this model can be reduced to one. Based on this model, a new fuel tank capacity method is derived that combines superior accuracy, ease of implementation, and robust predictive ability. A 95% mean prediction error confidence interval of −5.96 to 2.45% has been achieved with the new method. This improvement in accuracy enables a better representation of payload-range performance requirements and minimizes the demand for post-optimization fine-tuning.

Further improvement should focus on the development of a quantitative measure of the payload-range performance, to be incorporated either as an objective for designs with very diverse operational requirements or as a constraint to ensure that the optimized solutions possess realistic operational flexibility and growth potential. The improved Torenbeek method is expected to demonstrate even greater improvement if a larger sample can be found. It is also possible to refine the model with additional details such as a span limit of the outboard wing integral tanks or more advanced statistical techniques to reduce the negative impact of measurement errors.

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